Calcul de déterminants

```
/* Macro pour afficher un déterminant et sa valeur, factorisée.          */
cdet([m]) := block(
                [mat:substpart(MATRIX,m,0)],
                'determinant(mat)=factor(determinant(mat))
)$
```

```
> load("cdet.mc")$
> cdet([1,1,1],[a,b,c],[a^2,b^2,c^2]);
```

$$\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^2 & b^2 & c^2 \end{vmatrix} = (b-a)\,(c-a)\,(c-b)$$

```
> cdet([1,x,x],[x,1,x],[x,x,1]);
```

$$\begin{vmatrix} 1 & x & x \\ x & 1 & x \\ x & x & 1 \end{vmatrix} = (x-1)^2\,(2\,x+1)$$

```
> cdet([a,b,c],[b,c,a],[c,a,b]);
```

$$\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} = -(c+b+a)\left(c^2-b\,c-a\,c+b^2-a\,b+a^2\right)$$

```
> cdet([(b-c)^2,b^2,c^2],[a^2,(c-a)^2,c^2],[a^2,b^2,(a-b)^2]);
```

$$\begin{vmatrix} (b-c)^2 & b^2 & c^2 \\ a^2 & (c-a)^2 & c^2 \\ a^2 & b^2 & (a-b)^2 \end{vmatrix} = -2\,a\,b\,c\,(c-b-a)\,(c-b+a)\,(c+b-a)$$

```
> cdet([x,2,3],[3,x,2],[2,3,x]);
```

$$\begin{vmatrix} x & 2 & 3 \\ 3 & x & 2 \\ 2 & 3 & x \end{vmatrix} = (x+5)\left(x^2-5\,x+7\right)$$

```
> cdet([1+x,1,1],[1,1+x,1],[1,1,1+x]);
```

$$\begin{vmatrix} x+1 & 1 & 1 \\ 1 & x+1 & 1 \\ 1 & 1 & x+1 \end{vmatrix} = x^2\,(x+3)$$

```
> cdet([0,c,-b],[-c,0,a],[b,-a,0]);
```

$$\begin{vmatrix} 0 & c & -b \\ -c & 0 & a \\ b & -a & 0 \end{vmatrix} = 0$$

```
> cdet([0,m,m^2],[1/m,0,m],[1/m^2,1/m,0]);
```

$$\begin{vmatrix} 0 & m & m^2 \\ \frac{1}{m} & 0 & m \\ \frac{1}{m^2} & \frac{1}{m} & 0 \end{vmatrix} = 2$$

```
> cdet([0,sin(phi),sin(2*phi)],[sin(phi),0,sin(2*phi)],[sin(2*phi),sin(phi),0]);
```

$$\begin{vmatrix} 0 & \sin\varphi & \sin(2\varphi) \\ \sin\varphi & 0 & \sin(2\varphi) \\ \sin(2\varphi) & \sin\varphi & 0 \end{vmatrix} = \sin\varphi \sin(2\varphi) \left(\sin(2\varphi) + \sin\varphi\right)$$

```
> cdet([1,alpha,alpha^2],[beta,1,alpha],[beta^2,beta,1]);
```

$$\begin{vmatrix} 1 & \alpha & \alpha^2 \\ \beta & 1 & \alpha \\ \beta^2 & \beta & 1 \end{vmatrix} = (\alpha\beta - 1)^2$$

```
> cdet([-m,m-1,m],[2*m-1,m-1,-m],[-2,-4,2*m]);
```

$$\begin{vmatrix} -m & m-1 & m \\ 2m-1 & m-1 & -m \\ -2 & -4 & 2m \end{vmatrix} = -2\,(m-1)\,m\,(3m-1)$$

```
> cdet([1,1,1],[a,b,c],[a^3,b^3,c^3]);
```

$$\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^3 & b^3 & c^3 \end{vmatrix} = (b-a)\,(c-a)\,(c-b)\,(c+b+a)$$

```
> cdet([a+b,a*b,a^2+b^2],[b+c,b*c,b^2+c^2],[c+a,c*a,c^2+a^2]);
```

$$\begin{vmatrix} b+a & a\,b & b^2+a^2 \\ c+b & b\,c & c^2+b^2 \\ c+a & a\,c & c^2+a^2 \end{vmatrix} = (b-a)\,(c-a)\,(c-b)\,(b\,c+a\,c+a\,b)$$

```
> cdet([2*b,b-c-a,2*b],[a-b-c,2*a,2*a],[2*c,2*c,c-a-b]);
```

$$\begin{vmatrix} 2b & -c+b-a & 2b \\ -c-b+a & 2a & 2a \\ 2c & 2c & c-b-a \end{vmatrix} = -(c+b+a)^3$$