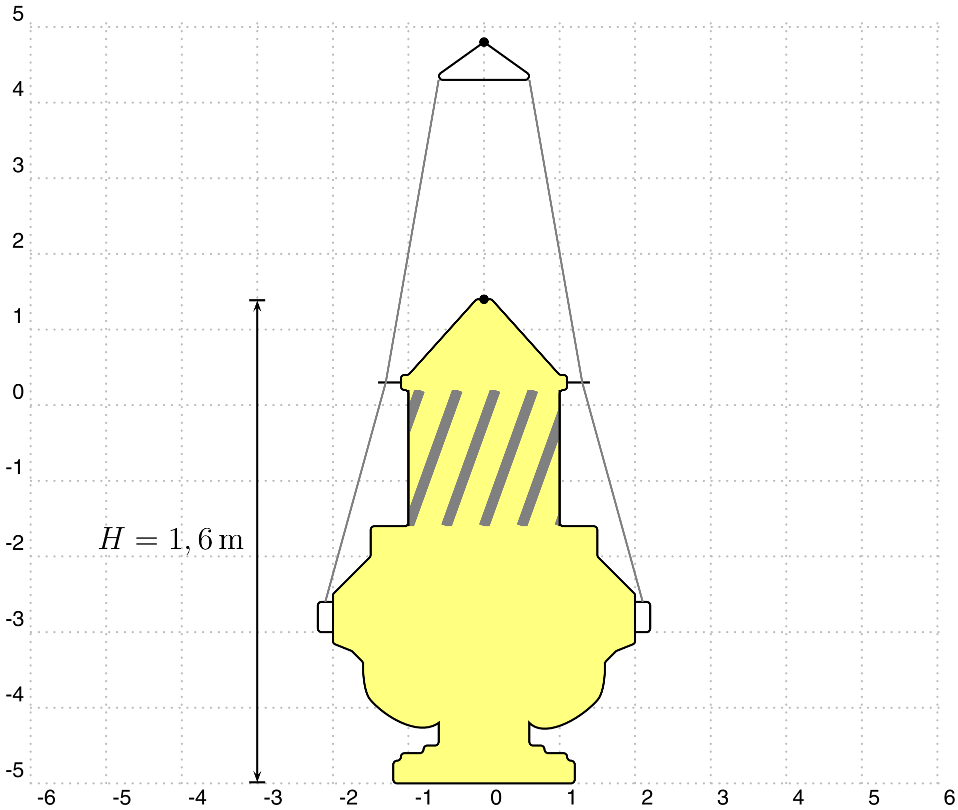
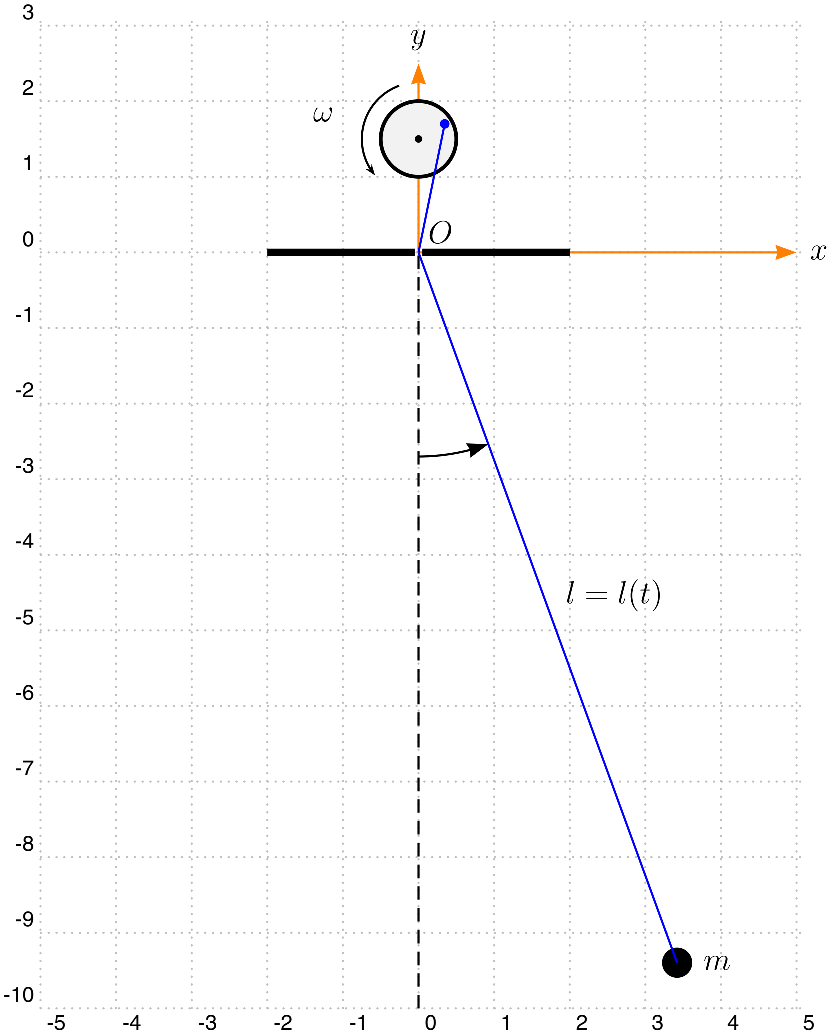
1 Botafumeiro – un pendule de longueur variable



Hauteur $H \approx$ 1,50-1,60 m
Masse $m \approx 54$ kg
Longueur de corde $l_0 \approx 30$ m
Amplitude $a \approx 2$ m (estimatif)



Les coordonnées de la masse m

$$x = l \sin \varphi$$
$$y = -l \cos \varphi$$

L'oscillation de la suspension ($a < l_0$):

$$l = l_0 + a \sin \omega t$$

Les derivées par rapport au temps :

$$\dot{x} = \dot{l} \sin \varphi + l\dot{\varphi} \cos \varphi$$
$$\dot{y} = -\dot{l} \cos \varphi + l\dot{\varphi} \sin \varphi$$
$$\dot{l} = a\omega \cos \omega t$$

L'énergie cinétique

$$\begin{aligned} T &= \frac{1}{2}m(\dot{x}^2 + \dot{y}^2) = \frac{1}{2}m(\dot{l}^2 + l^2\dot{\varphi}^2) \\ &= \frac{1}{2}ma^2\omega^2 \cos^2 \omega t + \frac{1}{2}m(l_0 + a \sin \omega t)^2\dot{\varphi}^2 \end{aligned}$$

L'énergie potentielle de pesanteur :

$$\begin{aligned} U &= mgy = -mgl \cos \varphi \\ &= -mg(l_0 + a \sin \omega t) \cos \varphi \end{aligned}$$

La fonction Lagrange :

$$
\begin{aligned}
L &= T - U \\
&= \frac{1}{2} m a^2 \omega^2 \cos^2 \omega t + \frac{1}{2} m (l_0 + a \sin \omega t)^2 \dot{\varphi}^2 + m g (l_0 + a \sin \omega t) \cos \varphi
\end{aligned}
$$

Pour φ, $\dot{\varphi}$, on recoit

$$
\begin{aligned}
\frac{\partial L}{\partial \varphi} &= -m g (l_0 + a \sin \omega t) \sin \varphi \\
\frac{\partial L}{\partial \dot{\varphi}} &= m (l_0 + a \sin \omega t)^2 \dot{\varphi} \\
\frac{\mathrm{d}}{\mathrm{d}t} \frac{\partial L}{\partial \dot{\varphi}} &= 2 m \omega \dot{\varphi} (l_0 + a \sin \omega t) \cos \omega t + m (l_0 + a \sin \omega t)^2 \ddot{\varphi}
\end{aligned}
$$

Avec du frottement $F = \alpha v$ on a la fonction de dissipation

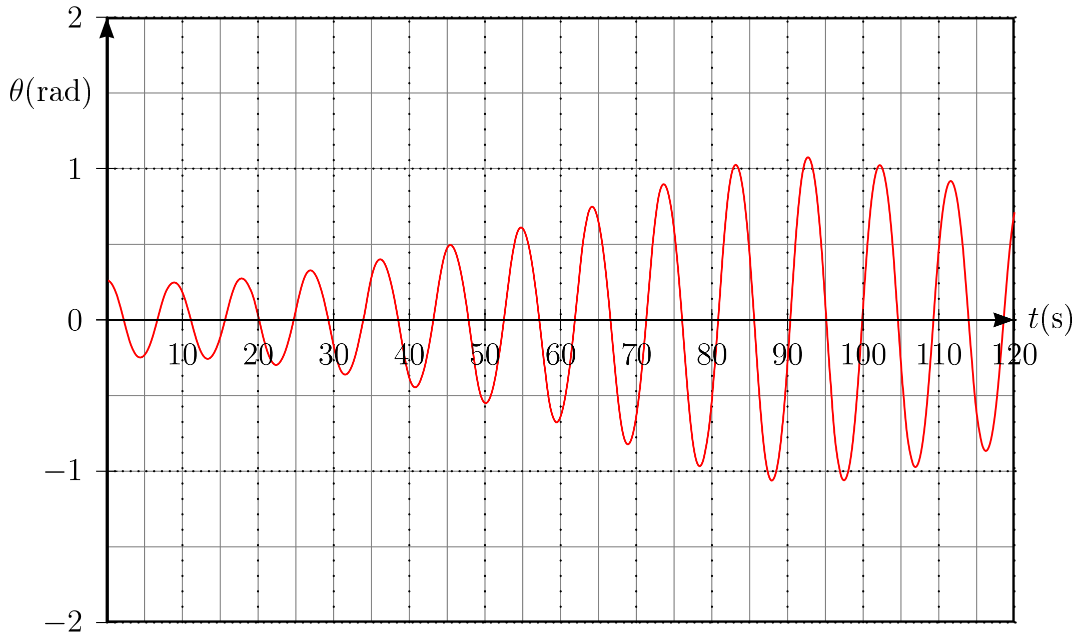
$$
D = \int_0^v h(v)\,\mathrm{d}v = \int_0^v \alpha v\,\mathrm{d}v = \frac{1}{2} \alpha v^2 = \frac{1}{2} \alpha (a^2 \omega^2 \cos^2 \omega t + (l_0 + a \sin \omega t)^2 \dot{\varphi}^2)
$$

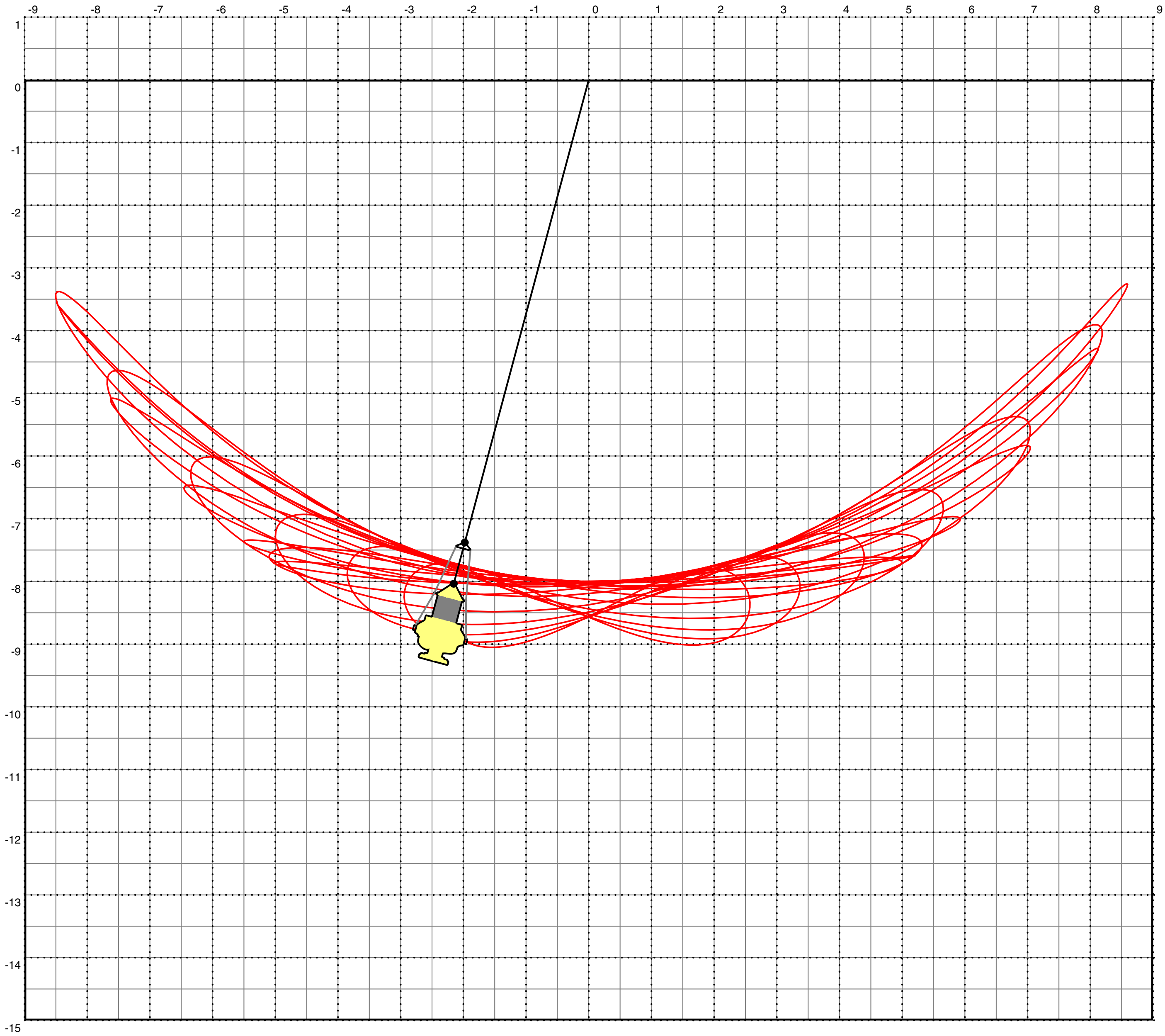
Le formalisme Lagrangienne donne:

$$
\frac{\mathrm{d}}{\mathrm{d}t} \frac{\partial L}{\partial \dot{\varphi}} - \frac{\partial L}{\partial \varphi} + \frac{\partial D}{\partial \dot{\varphi}} = 0
$$

L'équation différentielle :

$$
\boxed{\ddot{\varphi} + \frac{\alpha}{m} \dot{\varphi} + \frac{2 \omega \dot{\varphi} \cos \omega t + g \sin \varphi}{l_0 + a \sin \omega t} = 0}
$$




-9
-8
-7
-6
-5
-4
-3
-2
-1
0
1
2
3
4
5
6
7
8
9
1
0
-1
-2
-3
-4
-5
-6
-7
-8
-9
-10
-11
-12
-13
-14
-15